INSTITUTE FOR PRODUCTIVITY RESEARCH
488 Seventh Avenue
New York, NY 10018

Final Report 0001AE
Under Contract N00014-92-C-0101
Office of Naval Research

on project:

ADVANCED MODEL OF GRADUAL WEAR OF VISCOELASTIC SURFACE FILM

43pp

submitted to:

Office of Naval Research
Attn: Dr. Peter Schmidt
Materials Division/Code 1131
800 North Quincy Street
Arlington, VA 22217-5000

April, 1993

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FOREWORD

This final report of project, Advanced Model of Gradual Wear of Viscoelastic Surface Film, under Office of Naval Research Contract N00014-92-C-0101, contains three tabs:

Tab A Manuscript of the paper, "On the Prediction of Gradual Wear Life of Solid Lubricating Films;" this is to be presented at, and published by the 6th International Congress on Tribology, August 30-September 2, 1993, Budapest, Hungary.

Tab B Manuscript of the paper, "Theory of Gradual Wear Life Prediction of Viscoelastic Lubricating Surface Film;" this has been submitted for presentation at the 1993 Joint Tribology Conference, and publication in the STLE Tribology Transactions.

Tab C Report on a requisite task element of the contract: a method of verifying the results experimentally.

ON THE PREDICTION OF GRADUAL WEAR LIFE OF SOLID LUBRICATING FILMS[1]

by

Frederick F. Ling[2] and Wen-Ruey Chang[3]

Abstract

A simple model is proposed for the prediction of gradual wear life of solid lubricating films. The film is modeled physically as a viscoelastic material which obeys the general linear viscoelastic material constitutive law.

INTRODUCTION

The earliest recorded history placed the first use of solid lubricating film [1] in wheeled vehicles around 3000 B.C. As mankind approaches 2000 A.D., serious questions can be asked: "Where are we, scientifically, in terms of basic understanding of solid lubricating films?" While fluid-film lubrication theory had an earlier start in its development, there has been tremendous progress made in solid lubrication research in recent decades. The progress has been uneven, however, in terms of the various disciplinary facets which are needed to tell the whole story. Consequently we are not as advanced in our understanding of the whole as we could be because of the weakest link phenomenon. One of the weakest links is the area of mechanics of solid lubricating films [2-4]. This paper, therefore, is an attempt to address one of the deficiencies.

APPROACH

Rheology of Solid Lubricating Film

Rheological considerations of material involved surface phenomena might start with elastic-plastic materials [4-12]. A more substantial route has not been followed up since 1967; that is a general linear viscoelastic material as a surface layer in connection with deformation friction [13].

Proposed Model

Based on experimental evidences [3,4,14] and qualitative theoretical assessment [15], the following formulation is proposed.

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A. <u>Wear Criterion/Failure Rule</u> - Consider a thin film on a substrate with certain surface roughness as characterized by the r.m.s. value, σ_0. Consider, moreover, an initial film thickness, $h_0/\sigma_0 > 1$, say 2 or more. Consider the wear as the gradual loss of material which comes under a slider repeatedly. The slider is under a normal load P and it is in relative motion with the film/substrate material system. The slow diminution of material will form the wear criterion and the associated failure rule is that $h = h_f$, where h_f is the film thickness at failure; this is a parameter relating to $h_0/\sigma_0 -1$.

B. <u>Material System of the Model</u> - Consider a deformable cylinder with certain surface roughness, rotating at constant speed, ω. Let there be a thin film overlaying the cylinder. As indicated above, the initial film thickness is h_0.

The rationale is simply to capture the physics without unnecessarily making the analysis more complex than it needs to be. Various material constitutive relationships can be examined but, in this analysis, only the general linear viscoelastic material constitutive law is studied.

C. <u>Mechanics of the Proposed System</u> - Models outlined above have the repeated loading feature built in. As such, it is an order of magnitude more complex than most contact problems posed in the literature. The models also would show the complex nature of an analysis to bring quantitative measure of wear into consideration. The choice of viscoelastic favored over plastic material models has to do with the fact that the loading and unloading of plastic bodies, aside from the simplest geometry, is extremely difficult to handle. More importantly, recent research has shown, under certain physical situations, viscoelastic description may be more appropriate. In this formalism, the Newtonian framework on forces and stresses are set up independently from the material behavior. Material behaviors enter the problem through the formalism of constitutive relationship, *e.g.* stress-strain-strain rate law. The two sets of equations are then solved together with appropriate boundary and initial conditions.

There are still the questions of surface of the film and the slider geometry, among other factors, to be decided before a mechanics problem can be properly posed. We propose a uniform distribution of load under the slider. Also assumed is the negligible shear traction on the surface as we are addressing a solid lubricating system. The problem as posed is a transient problem as opposed to the usual steady state or quasi-steady state problems in tribology. The film thickness as a function of time must be found; this connotes wear rate. Also, the time at which the film thickness has diminished to a critical point must be found; this connotes failure. Of course, failure time determines wear life and the analysis is expected to provide wear life prediction.

GOVERNING EQUATIONS

The general linear viscoelastic material in two dimensions is characterized by

$$P\sigma_{ij} = Q\epsilon_{ij} + \left[\kappa P - \frac{Q}{3}\right] \delta_{ij}\, \epsilon_{kk}\,, \qquad (1)$$

where i,j = 1,2; repeated indices denote summation; σ_{ij} is the stress tensor, ε_{ij} is the strain tensor and δ_{ij} is the Kronecker delta; K is the bulk modulus;

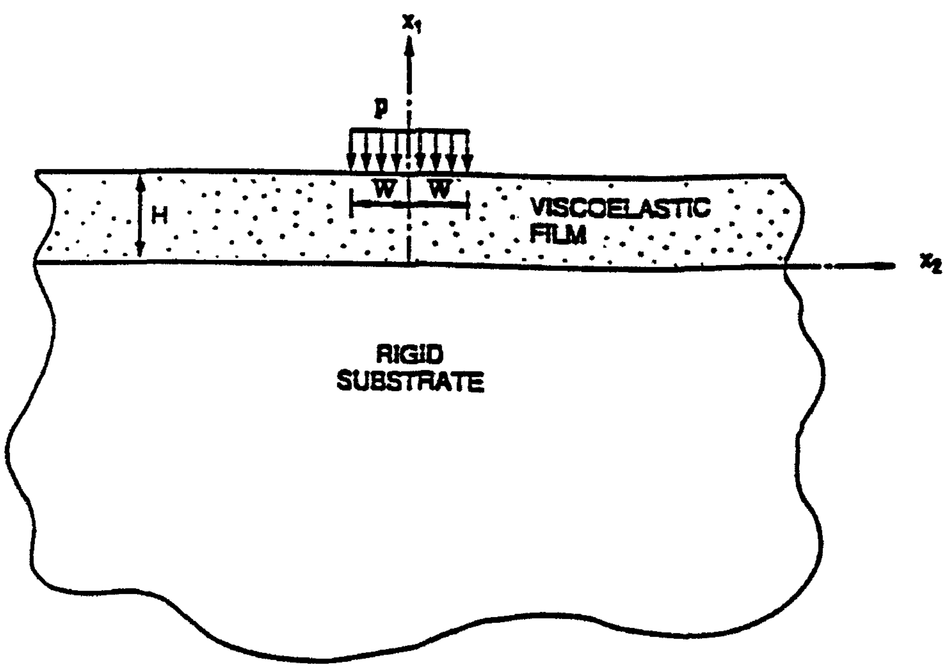
$$P = \frac{\partial^2}{\partial t} + \alpha \frac{\partial}{\partial t} + \beta \qquad \text{and} \qquad Q = \gamma \frac{\partial^2}{\partial t^2} + \Delta \frac{\partial}{\partial t}\,,$$

in which α, β, γ and Δ are materials constants to be enumerated later.

The equations of equilibrium in two dimensions are

$$\sigma_{ij,j} = 0. \qquad (2)$$

The figure shows the geometry of the problem to be solved. That is, the model is that of the cross-section of the viscoelastic film on a rigid substrate under load, which is spread over the width 2W, and perpendicular to the plane of the paper, a length L, for an instant of time. p denotes the normal load per unit area.



METHOD OF SOLUTION / SIMPLE THEORY

Equations (1) and (2) with the appropriate boundary and initial conditions constitute a complex initial value problem which requires extensive numerical computation. For the purpose of this paper, however, we will solve only the problem for the simple theory. That is, the problem as posed, but with $W \to \infty$. Thus, equations (1) become

$$\ddot{F} + \alpha \dot{F} + \beta F = \gamma \ddot{\delta} + \Delta \dot{\delta}\,, \tag{3}$$

where F is the normal stress and δ is normal strain.

Note, in the simple theory, equilibrium equations (2) are satisfied identically. As such equation (3) needs to be solved with the appropriate boundary and initial conditions. Space limitation does not permit steps to be listed explicitly in the text. In any event, we involve Laplace transform to equation (3), and solve the resulting equation for the strain δ. Upon inverse Laplace transform the following was obtained. That is, at the end of the first cycle, $t = \frac{2\pi}{\omega}$, in which ω is the rotational speed of our mechanical model that is a cylinder of radius R,

$$\delta\left(\frac{2\pi}{\omega}\right) = -\frac{L\beta}{\omega R \Delta} - \frac{e^{-\frac{\Delta}{\gamma}\cdot\frac{2\pi}{\omega}}}{\gamma}\left[1 - e^{\frac{\Delta L}{\gamma \omega R}}\right]\left[1 - \alpha\frac{\gamma}{\Delta} + \beta\left(\frac{\gamma}{\Delta}\right)^2\right]. \tag{4}$$

Note equation (4) is for a unit load F; so for a given load, equation (4) must be multiplied by the load. $H\delta$ with δ according to equation (4) provides the net displacement after the first cycle. As such the film thickness after this cycle, and at the beginning of the second cycle, is $H(1-\delta)$. The net displacement after the second cycle is then $H(1-\delta)\delta$. Accordingly, at the outset of the third cycle, the film thickness is $H(1-\delta) - H(1-\delta)\delta$ or $H(1-\delta)^2$. Generalizing, the film thickness after the nth cycle is $H(1-\delta)^n$. It is this thickness that has to be equated to a thickness at failure, discussed under Item A of Proposed Model, h_f

$$h_f = H(1-\delta)^n \tag{5}$$

with h_f and H known, δ from equation (4), n can be calculated. Of course, n is the gradual wear life as measured in terms of cycles. Again, δ needs to be multiplied by the load, say P for the simple model.

Numerical Example

Let h_f be twice the r.m.s. value of the surface roughness of the substrate, σ_o; the equations to solve from equations (5) and (4) are:

$$n = \log \frac{h_f}{H} / \log(1-\delta_p)\,, \tag{6}$$

where

$$\delta_p(\tfrac{2\pi}{\omega}) = P\left\{\frac{L\beta}{\omega R\Delta} + \frac{e^{-\frac{\Delta}{\gamma}\cdot\frac{2\pi}{\omega}}}{\gamma}\left[1 - e^{\frac{\Delta L}{\gamma\omega R}}\right]\cdot\left[1 - \alpha\frac{\gamma}{\Delta} + \beta\left(\frac{\gamma}{\Delta}\right)^2\right]\right\}. \tag{7}$$

Let the material be polyisobutylene (PIB), for which experimental data exist. There is a dearth of data on solid lubricants; thus the choice of PIB is for illustration only of the capability of the model as proposed. The data extracted from the literature [16,17] is linearized in the frequency of excitation comparable to the speed that is to be chosen for this example, *i.e.*, 1,000 r.p.m. The extracted data read: $\alpha = 4.288 \times 10^3 s^{-1}$, $\beta = 2.583 \times 10^5 s^{-2}$, $\gamma = 2.799 \times 10^6$ pascal, $\Delta = 1.963 \times 10^9$ pascal $\cdot s^{-1}$, $\kappa = 4 \times 10^9$ pascal.

We further select the following geometrical parameters: $L = 6.350 \times 10^{-3}$m, $H = 1 \times 10^{-5}$m, $\delta_o = 1 \times 10^{-6}$m, $R = 2 \times 10^{-2}$m. Table 1 shows predicted values of n from equation (6), based on δ_p from equation (7) for three values of P.

Table 1. Predicted Values of Gradual Wear Life, n

P(p.s.i.)	P(pascals)	δ_p	n(cycles)
0.1	69	2.748×10^{-5}	58,540
0.5	345	1.375×10^{-4}	11,708
1.0	689	2.748×10^{-4}	5,854

CONCLUSION

We believe the model adopted for the predictic.: of gradual wear life is plausible. Although, in this paper, only the simplified model is evaluated numerically, the numbers do lend credence to the suppositions which made up the physical model. Since we are not aware of any model like this one in the literature, we are pleased to offer the simple model to the readership. Of course, the more comprehensive model, based on equations (1) and (2) has yet to be evaluated in a quantitative way. In the meantime, there is a need for experimental data on the relevant material properties indicated. We hope both the quantitative evaluation of equations (1) subject to equations (2), and the availability of relevant material properties of solid lubricants will become a reality in the not too distant future.

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THEORY OF GRADUAL WEAR LIFE PREDICTION OF VISCOELASTIC LUBRICATING SURFACE FILM[1]

by

Wen–Ruey Chang[2] and Frederick F. Ling[3]

ABSTRACT

Solid lubricating surface film is modeled by general linear viscoelastic material. The appropriate governing equations are solved for the displacement of such a thin film under repeated loadings. The predicted transient motion of the surface is monitored. The gradual diminution of the lubricant film thickness under repeated loadings is considered as the film wear. The failure of solid lubricating film occurs when the existing thickness is to the same order of magnitude as the substrate surface roughness.

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INTRODUCTION

The earliest recorded history placed the first use of solid lubricating film [1] in wheeled vehicles around 3000 B.C. As mankind approaches 2000 A.D., serious questions can be asked: "Where are we, scientifically, in terms of basic understanding of solid lubricating films?" While fluid–film lubrication theory had an earlier start in its development, there has been tremendous progress made in solid lubrication research in recent decades. The progress has been uneven, however, in terms of the various disciplinary facets which are needed to tell the whole story. Consequently we are not as advanced in our understanding of the whole as we could be because of the weakest link phenomenon. One of the weakest links is the area of mechanics of solid lubricating films [2–4].

The reasons for the unevenness in progress are many, not the least of which is the horrendous complexity of the subject area. Figure 1 illustrates one recent view of the complexity [5]. From the application point of view, there is a black box between input and output, the dash–and–dot box. Ideally, given the input, one can query the output, e.g. information on wear. In reality, the aforementioned black box contains a whole host of attributes which need scientific answers. It is the completeness of answers which gives integrity and substance to the black box as to its usefulness in fulfilling its function. And it is fair to say that much of the basic research efforts over the last five decades have been devoted to many tribological processes, with attending efforts for the understanding of the great variety of properties and, of course, of many many configurations of structural elements.

Again, of the five regimes cited in the solid black box in Figure 1, solid lubrication as a system study has received the least cumulative attention. In this regard a recent National Science Foundation symposium concluded on research needs in tribology [6] this way: "...one area stands out...establishment of models...future prediction...tribological components and systems."

For the practical applications, a design engineer would like to estimate the longevity and the proper thickness of the thin film solid lubricants. The development of fundamental understanding and life prediction model will also help estimate the duration of each maintenance cycle or frequency for replenishment of the film.

The current study will address both the fundamental understanding and the practical aspects of the thin film solid lubricants.

APPROACH

Rheology of Solid Lubricating Film

Rheological considerations of material involved surface phenomena might start with elastic–plastic materials [4,7–12]. A more substantial route has not been followed up since 1967; that is a general linear viscoelastic material as a surface layer in connection with deformation friction [13]. Figure 2, from reference [13], shows that contributions to deformation friction for an incremental time change of Δt or motion: those shown in REGION I. The changes in REGION II represent unloading which becomes heated and the heat is then dissipated.

Proposed Model

This model, complete with its simplifications, is originated with the proviso that strictly computational modeling based on well understood field equations will be outside the province of this study. Needless to say, however, it is expected that there will be a need for computational work towards the end of the establishment of the theoretical model. Based on experimental evidences [3,4,14] and qualitative theoretical assessment [15,16,17], the following formulation is proposed.

A. Wear Criterion/Failure Rule – Consider a thin film on a substrate with certain surface roughness as characterized by the r.m.s. value, σ_0. Consider, moreover, an initial film thickness, $h_0/\sigma_0 > 1$, say 2 or more. Consider the wear as the gradual loss of material which comes under a slider repeatedly. The slider is under a normal load P and it is in relative motion with the film/substrate material system. The slow diminution of material will form the wear criterion and the associated failure rule is that $h = h_f$, where h_f is the film thickness at failure; this is a parameter relating to $h_f/\sigma_0 \simeq 1$.

B. <u>Material System of the Model</u> – Consider a deformable cylinder with certain surface roughness, rotating at constant speed, ω. Let there be a thin film overlaying the cylinder. As indicated above, the initial film thickness is h_0. The rationale is simply to capture the physics without unnecessarily making the analysis more complex than it needs to be. Various material constitutive relationships can be examined but, in this analysis, only the general linear viscoelastic material constitutive law will be studied.

C. <u>Mechanics of the Proposed System</u> – Models outlined above have the repeated loading feature built in. As such, it is an order of magnitude more complex than most contact problems posed in the literature. The models also would show the complex nature of an analysis to bring quantitative measure of wear into consideration. The choice of viscoelastic favored over plastic material models has to do with the fact that the loading and unloading of plastic bodies, aside from the simplest geometry, is extremely difficult to handle. More importantly, recent research has shown, under certain physical situations, viscoelastic description may be more appropriate. The point to note is that plastic and viscoelastic are not mutually exclusive! With several material properties, the general linear viscoelastic model gives a relative measure of the viscousness and a relative measure of elasticity of the material. Anisotropic effects as well as microstructural effects all enter through the constitutive relationship formulation. In this formalism, the Newtonian framework on forces and stresses are set up independently from the material behavior. Material behaviors enter the problem through the formalism of constitutive relationship, e.g. stress–strain– strain rate law. The two sets of equations are then solved together with appropriate boundary and initial conditions.

There are still the questions of surface of the film and the slider geometry, among other factors, to be decided before a mechanics problem can be properly posed. We propose a uniform distribution of load under the slider. Also assumed is the negligible shear traction on the surface as we are addressing a solid lubricating system. The problem as posed is a transient problem as cpposed to the usual steady state or quasi–steady state problems. The film thickness as a function of time must be found; this connotes wear rate. Also, the time at which the film thickness has diminished to a critical point must be found; this connotes failure. Of course, failure time determines wear life and the analysis is expected to provide wear life prediction. The associated stress fields, strain–rate (flow) fields and displacements on the surface will be part of the solution sought.

GOVERNING EQUATIONS

We further assume that this model takes into account two spatial dimensions in addition to the temporal variable.

One of the spatial dimensions is in the thickness direction of the film and the second spatial dimension is in the direction transverse to the sliding direction. It is assumed that the plane strain condition is valid in the plane defined by these two dimensions. Figure 3 shows the geometry of the problem to be solved. That is, the model is that of the cross–section of the viscoelastic film on a rigid substrate under load, which is spread over the width 2W and perpendicular to the plane of the paper with length L for an instant of time. Let p denote the normal load per unit area.

The Airy's stress function is suitable for solving a plane problem with a mixed boundary condition such as the one that will be posed here. Recall the Airy's stress function representation [18]

$$\sigma_{11} = \frac{\partial^2 \psi}{\partial x_2{}^2}$$

$$\sigma_{22} = \frac{\partial^2 \psi}{\partial x_1{}^2} \tag{1}$$

$$\sigma_{12} = -\frac{\partial^2 \psi}{\partial x_1 \partial x_2}$$

where ψ is the Airy's stress function. The Airy's stress function automatically satisfies the equation of equilibrium,

$$\sigma_{ij,j} = 0 \tag{2}$$

where i = 1, 2; repeated indices denote summation. The equation of compatibility requires the Airy's stress function to satisfy

$$\nabla^2(\nabla^2\psi) = 0\ , \tag{3}$$

where ∇^2 is the Laplacian operator.

The general linear viscoelastic material in two dimensions is characterized by

$$P\sigma_{ij} = Q\ \epsilon_{ij} + \left[\ \kappa P - \frac{Q}{3}\right]\ \delta_{ij}\epsilon_{kk}\ , \tag{4}$$

where κ is the bulk modulus; and

$$P = \frac{\partial^2}{\partial t^2} + \alpha \frac{\partial}{\partial t} + \beta$$

$$Q = \gamma \frac{\partial^2}{\partial t^2} + \Delta \frac{\partial}{\partial t}, \tag{5}$$

in which α, β, γ and Δ are material constants to be enumerated later. The strain can be obtained from the inverse of equation (4), i.e.,

$$Q\, \epsilon_{ij} = P\, \sigma_{ij} - \frac{1}{3\kappa}\left[\kappa P - \frac{Q}{3} \right] \delta_{ij}\sigma_{kk}\,. \tag{6}$$

The strain–displacement relationship for viscoelastic material is identical to that for elastic material, i.e.,

$$\epsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right], \tag{7}$$

where u_1 and u_2 are the displacements in the x_1 and x_2 directions, respectively. The single–valued displacement functions can be obtained from the path integration of the strain and the strain–displacement relationship, equation (7), since the stress function satisfies the equation of compatibility, equation (3).

Define the following nondimensional quantities:

$z_1 = x_1/W$, $z_2 = x_2/W$, $h = H/W$, $v_1 = u_1/W$, $v_2 = u_2/W$, $q = p/\kappa$, $\tau_{11} = \sigma_{11}/\kappa$, $\tau_{22} = \sigma_{22}/\kappa$, $\tau_{12} = \sigma_{12}/\kappa$ and $\phi = \psi/\kappa W^2$.

At any instant, the film–substrate combination, which is under the load moving in a direction perpendicular to the paper at constant speed, the following boundary conditions must prevail:

$$\tau_{11} = \begin{cases} -\ q[u(t-t_0) - u(t-t_0-\delta_t)] & (z_1 = h,\ |z_2| \le 1) \\ 0 & (z_1 = h,\ |z_2| > 1) \end{cases}$$

$$\tau_{12} = 0 \qquad (z_1 = h,\ |\rho| < \infty) \tag{8}$$

$$v_1 = v_2 = 0 \qquad (z_1 = 0,\ |\rho| < \infty)\ .$$

where u(t) is the Heaviside unit step function. The regularity conditions τ_{11}, τ_{12}, τ_{22}, v_1, v_2 $\to 0$ for $0 \le z_1 \le h$ and $(z_1^2 + z_2^2)^{1/2} \to \infty$.

METHOD OF SOLUTION

The Laplace transform and the Fourier transform will be used to solve the problem defined by equations (1)–(8). Recall the Laplace transform and its inverse

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt$$

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{f}(s)e^{st}ds\ .$$

Apply the Laplace transform to the problem and assume that all the initial conditions are zero. After the Laplace transform, a viscoelastic problem can thus be treated like an elastic problem.

The Airy's stress function representation, equation (1), can be rewritten as

$$\bar{\tau}_{11} = \frac{\partial^2 \bar{\phi}}{\partial z_2{}^2}$$

$$\bar{\tau}_{22} = \frac{\partial^2 \bar{\phi}}{\partial z_1{}^2} \tag{9}$$

$$\bar{\tau}_{12} = -\frac{\partial^2 \bar{\phi}}{\partial z_1 \partial z_2}\ .$$

Equation (3) can be rewritten as

$$\nabla^2(\nabla^2\bar{\phi}) \equiv 0\ , \tag{10}$$

or

$$\frac{\partial^4\bar{\phi}}{\partial z_1{}^4} + 2\,\frac{\partial^4\bar{\phi}}{\partial z_1{}^2\partial z_2{}^2} + \frac{\partial^4\bar{\phi}}{\partial z_2{}^4} = 0\ . \tag{11}$$

Recall Fourier transform and its inverse

$$\tilde{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-i\xi x}dx$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\xi)e^{i\xi x}d\xi\ .$$

Apply the Fourier transform in the z_2 direction. After two transformations, the stress–strain relationship, equation (4), can be non–dimensionalized and be rewritten as

$$\begin{aligned} \kappa(s^2 + \alpha s + \beta)\tilde{\bar{\tau}}_{ij} &= (s^2\gamma + s\Delta)\tilde{\bar{\epsilon}}_{ij} \\ &\quad + [\kappa(s^2 + \alpha s + \beta) - \frac{1}{3}(s^2\gamma + s\Delta)]\delta_{ij}\tilde{\bar{\epsilon}}_{kk}\ , \end{aligned} \tag{12}$$

The strain– displacement relationship in non–dimensional terms, equation (7), becomes

$$\tilde{\bar{\epsilon}}_{11} = \frac{\partial\tilde{\bar{v}}_1}{\partial z_1},\ \ \tilde{\bar{\epsilon}}_{12} = \frac{1}{2}\,(\frac{\partial\tilde{\bar{v}}_2}{\partial z_1} + i\xi\tilde{\bar{v}}_1),\ \ \tilde{\bar{\epsilon}}_{22} = i\xi\tilde{\bar{v}}_2\ . \tag{13}$$

The boundary conditions, equation (8), become

$$\begin{aligned}
&\tilde{\tilde{\tau}}_{11} = -\frac{2q\ \sin\ \xi}{\xi} \cdot \frac{e^{-st_0}(1 - e^{-s\delta_t})}{s} && , \ z_1 = h \\
&\tilde{\tilde{\tau}}_{12} = 0 && , \ z_1 = h \qquad (14) \\
&\tilde{\tilde{v}}_1 = \tilde{\tilde{v}}_2 = 0 && , \ z_1 = 0 \, .
\end{aligned}$$

The equation of compatibility, equation (11), becomes

$$\frac{d^4\tilde{\tilde{\phi}}}{dz_1^{\,4}} - 2\xi^2 \frac{d^2\tilde{\tilde{\phi}}}{dz_1^{\,2}} + \xi^4\tilde{\tilde{\phi}} = 0 \, . \qquad (15)$$

The general solution to equation (15) is

$$\tilde{\tilde{\phi}} = (A + Bz_1)\sinh(\xi z_1) + (C + Dz_1)\cosh(\xi z_1) \, . \qquad (16)$$

The displacements can be obtained from the Airy stress function as shown in [18]. The coefficients, A, B, C and D, can be obtained from the boundary conditions, equation (14). Since the normal displacement at the surface is the amount of wear incurred, it is the quantity of the greatest interest and can be obtained in the form of

$$\tilde{\tilde{v}}_1 = \frac{4q\ \sin\ \xi}{\xi^2} \cdot \frac{e^{-st_0}(1 - e^{-s\delta_t})}{s} \cdot \Lambda(\Lambda + \tfrac{2}{3}) \cdot$$

$$\frac{(2\Lambda + \frac{1}{3})h\xi - (\Lambda + \frac{7}{6})\sinh(2\xi h)}{(2\Lambda + \frac{1}{3})^2(h\xi)^2 + 4(\Lambda + \frac{2}{3})^2\cosh^2(\xi h) - \sinh^2(\xi h)} , \qquad (17)$$

where $\Lambda = \dfrac{\kappa(s^2 + \alpha s + \beta)}{s^2\gamma + s\Delta}$.

The actual surface displacement can be obtained after the inverse Fourier transform and the inverse Laplace transform are performed sequentially on equation (17). Due to the complexity of the problem, both inverse transforms are done numerically. The numerical inverse Fourier transform can be done by using fast Fourier transform (FFT) [19]. IMSL subroutine INLAP is used for the inverse Laplace transform. To preserve the sequence of the inverse transformations, the inverse Fourier transform should be performed before the inverse Laplace transform.

The numerical inverse Fourier transform can be done easily after equation (17) is properly constructed in the transformed domain. A much greater difficulty arises in the inverse Laplace transform where all of the singularities (poles) need to be located for the proper choice of the integration path. This implies that a search for all of the poles in the complex plane needs to be conducted for every discrete point in the inverse Fourier transform. The situation is further complicated by the fact that the total number of the poles in the inverse Laplace transform is not easily known from equation (17). The effort to search for the poles in the inverse Laplace transform can be tremendous and incomplete.

An alternative approach, henceforth referred to as the secc ·d approach in distinction from the previous (the first) approach, is to reverse the sequence of the inverse transformations, i.e., the inverse Laplace transform first and then the inverse Fourier transform afterward. The real advantage of this approach is that the direct pole search on equation (17) for the inverse Laplace transform is much easier than the previous approach. Upon examining equation (17), one can easily conclude that there exists six (6) poles for each ξ value. Besides the poles at (0,0) and $(-\Delta/\gamma, 0)$, the other four (4) can be determined from the denominator of equation (17). The search for the poles is identical to the search for the roots of the equation

$$
\begin{aligned}
&[2\kappa\,(s^2 + \alpha s + \beta) + \tfrac{1}{3}\,(s^2\gamma + s\Delta)]^2\,(h\xi)^2 + \\
&[2\kappa\,(s^2 + \alpha s + \beta) + \tfrac{4}{3}\,(s^2\gamma + s\Delta)]^2 \cosh^2(h\xi) \\
&\qquad - (s^2\gamma + s\Delta)^2 \sinh^2(h\xi) = 0\,. \qquad (18)
\end{aligned}
$$

RESULTS AND DISCUSSION

Since the upper bound of the numerical integration of the inverse Fourier transform is finite, the pole search will be carried out up to a finite ξ value only. The polyisobutylene (PIB) film will be used as an example in this paper. The measurement data of the shear compliance of the PIB film are available in the literature [20]. The material properties for a four–element viscoelastic model is further deduced in [21] from the measurements reported in [20]. For viscoelastic material, the material properties are frequency dependent. The material properties in the low frequency range (30–300 Hz) in [21] will be used. The constitutive equation for a pure shear can be obtained from a more general two–dimensional constitutive equation, equation (4). However, modification of material constants are needed to extrapolate a one dimensional measurement to a more general two dimensional model. The modified material properties, the mechanical dimensions and the loading on the film are listed in Table 1.

The pole search for the inverse Laplace transform is carried out for ξ value from zero (0) to 200π. The negative branch is omitted because of symmetry. The pole search shows that all the poles are located at the negative real axis and the origin. Since the thickness of the coating will be reduced because of wear, the pole search does not need to be repeated as long as the upper bound of the inverse Fourier transform is unchanged.

For the pole locations in the inverse Laplace transform of the first approach, the right hand side of equation (17) is divided into four parts by the dot. The first part contributes most of the fluctuation in the inverse Fourier transform and decays rapidly. Within the finite range of integration for the inverse Fourier transform, the fourth part of equation (17) is almost unchanged since ξ is multiplied by h, a small value, and the first part decays rapidly. Therefore, the pole location can almost be determined by the fourth part of equation (17). It is suggested that the poles for the inverse Laplace transform in the first approach might be in the vicinity of the pole locations for the inverse transform in the second approach. Although there are six poles for every ξ value in the first approach, it is not suggested that the number of the poles in the first approach is identical to the number of the poles in the second approach. However, the second approach will be used for the results presented. The first approach will be used, with the assumption that the poles for the inverse Laplace transform are in the vicinity of the poles for the inverse Laplace transform in the second approach, to verify some of the results obtained by the second approach since the pole search for the inverse Laplace transform in the second approach is more vigorous and more complete than the pole search in the first approach.

The surface normal displacement during the first loading cycle is shown in Figures 4, 5 and 6. Figure 4 contains the results of the surface normal displacement at the center of the loading region ($z_2 = 0$) during the loading period. When the load is suddenly applied at $t = 0$, there is an instantaneous elastic displacement. There is a monotonously gradual increase in displacement due to the viscous effect when the load remains applied. When the load is suddenly removed at $t = 1 \times 10^{-4}$ sec, there is an instantaneous elastic recovery and the gradual viscous recovery begins. The viscoelastic recovery over a longer period of time compared with the loading period is shown in Figure 5. Right after the load is removed, there is a rapid recovery. The rate of recovery gradually diminishes as the time increases. The surface normal displacement over the whole loading region is shown in Figure 6. Due to the symmetry, only half of the region is shown. The results indicate that the surface normal displacement is quite uniform over the entire loading region except near the edges. Some of the roundness near the edge is real, nevertheless some is due to the numerical error introduced by the numerical inverse Fourier transform (FFT).

The quantity of particular interest in this investigation is the life of film. The life of film is determined by the film thickness and the failure criterion. The film is under repeated loadings. In this study, it is assumed that the frequency of loading is 60 Hz. As shown in Figure 5, there is a long recovery after each loading. Keeping track of the recovery of each loading throughout the entire film life could be a numerical nightmare. A simplified approach would be to ignore the further recovery when the following loading arrives. As shown in Figure 5, most of the recovery has been accounted for when the next loading arrives ($t = 0.0167$ sec). This simplification greatly reduces the numerical effort involved. The average reduction in film thickness when the following loading arrives is considered as the wear of film. The film thickness is reduced in every loading. Different film thickness will result in a different amount of wear. Another problem is that the film thickness needs to be adjusted for every loading cycle in order to account for wear correctly. This means that the modeling of film wear needs to be performed for each loading throughout the entire film life. The computation could be enormous for a long–lasted film. A simplification would be to adjust the thickness once in every certain number of loading cycles. The number of loading cycles for the thickness adjustment highly depends on the longevity of the film. A long–lasted film allows a larger number of cycles while a film with a very short life could require the modeling of each individual loading.

The other issue of life prediction is the failure criterion. One of the established criteria might be friction. The failure of the solid lubricating film is considered as when the friction increases significantly. Friction is a global measurement of the resistance to sliding which can be affected by many factors and, therefore, is not a well–defined terminology for failure. All the engineering surfaces are rough. A good measurement of failure could be the remaining film thickness compared with the surface roughness of the substrate. The failure criterion could be when the remaining film thickness is the same order of magnitude as the substrate surface roughness. When high asperities do not have enough lubricant on the surface, this could lead to high friction. In this study, it is assumed that the film failure occurs when the remaining film thickness is two times the root–mean–square (r.m.s.) of the substrate surface roughness, σ_0.

The life prediction results based on the film wear model and the failure criterion described previously are shown in Figure 7. The film thickness is adjusted once every one thousand (1000) loading cycles. The predicted life of film is 1.4×10^6 cycles. The wear rate of film is high at the early stage of life and then gradually reduces. A curve can be used to approximate the predicted results

$$h = h_0 \left[1 - \frac{\Delta h}{h_0} \right]^{0.2n} \tag{19}$$

where h is the film thickness after n cycles, h_0 is the initial film thickness and Δh is the film wear in the first loading cycle when $h = h_0$. The error due to curve fitting is less than 0.5%.

CONCLUSION

Solid lubricating film is modeled by general linear viscoelastic material. The Airy's stress function, Laplace transform and Fourier transform are used to solve the surface film problem with the mixed boundary condition. The predicted transient motion of the surface normal displacement is monitored. In each loading cycle, there is an instantaneous elastic displacement when the load is applied and a monotonously increase in displacement due to the viscous effect when the load remain applied. The recovery is rapid right after the loading period and diminishes gradually as the time increases. Further recovery is ignored when the following loading arrives. The gradual diminution of the lubricant film thickness under repeated loadings is considered as the film wear. The failure criterion of film is defined as when the remaining film thickness is to the same order of magnitude as the substrate surface roughness.

PIB film is used as an example to illustrate this life prediction model. The published measurement of material properties in one dimension is modified for a more general two dimensional model. The transient motion of surface normal displacement and the remaining film thickness over the entire film life are presented.

This is an initial attempt to solve a very complicated problem. Further improvement in the viscoelastic model for solid lubricating film, measurement of material properties, investigation of failure criterion and the experimental verification of the model are the essential parts of the further development of this model.

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Table 1

Material Constants and System Parameters

α	=	4.287×10^{3} sec^{-1}
β	=	1.200×10^{5} sec^{-2}
γ	=	2.048×10^{11} Pascal
Δ	=	1.655×10^{7} Pascal/sec
κ	=	4.000×10^{9} Pascal
q	=	2.758×10^{-3}
h_o	=	3.1496×10^{-3}
t_o	=	0 sec
δ_t	=	1.0×10^{-4} sec
H	=	1.0×10^{-5} m
W	=	3.175×10^{-3} m
σ_o	=	1.0×10^{-6} m

Figure Captions

Figure 1.	Schematic Diagram Showing Global Input–Output System in Tribology and Its Complex Sub–Systems.
Figure 2.	Schematic Diagram Depicting Deformation Friction of Viscoelastic Surface Films.
Figure 3.	Schematic Diagram of the Model Details.
Figure 4.	Surface Normal Displacement at the Center of Loading Region During the Loading Period.
Figure 5.	Surface Normal Displacement at the Center of Loading Region After the Loading Period.
Figure 6.	Surface Normal Displacement Over the Entire Loading Region.
Figure 7.	The Remaining Film Thickness Over the Entire Film Life.

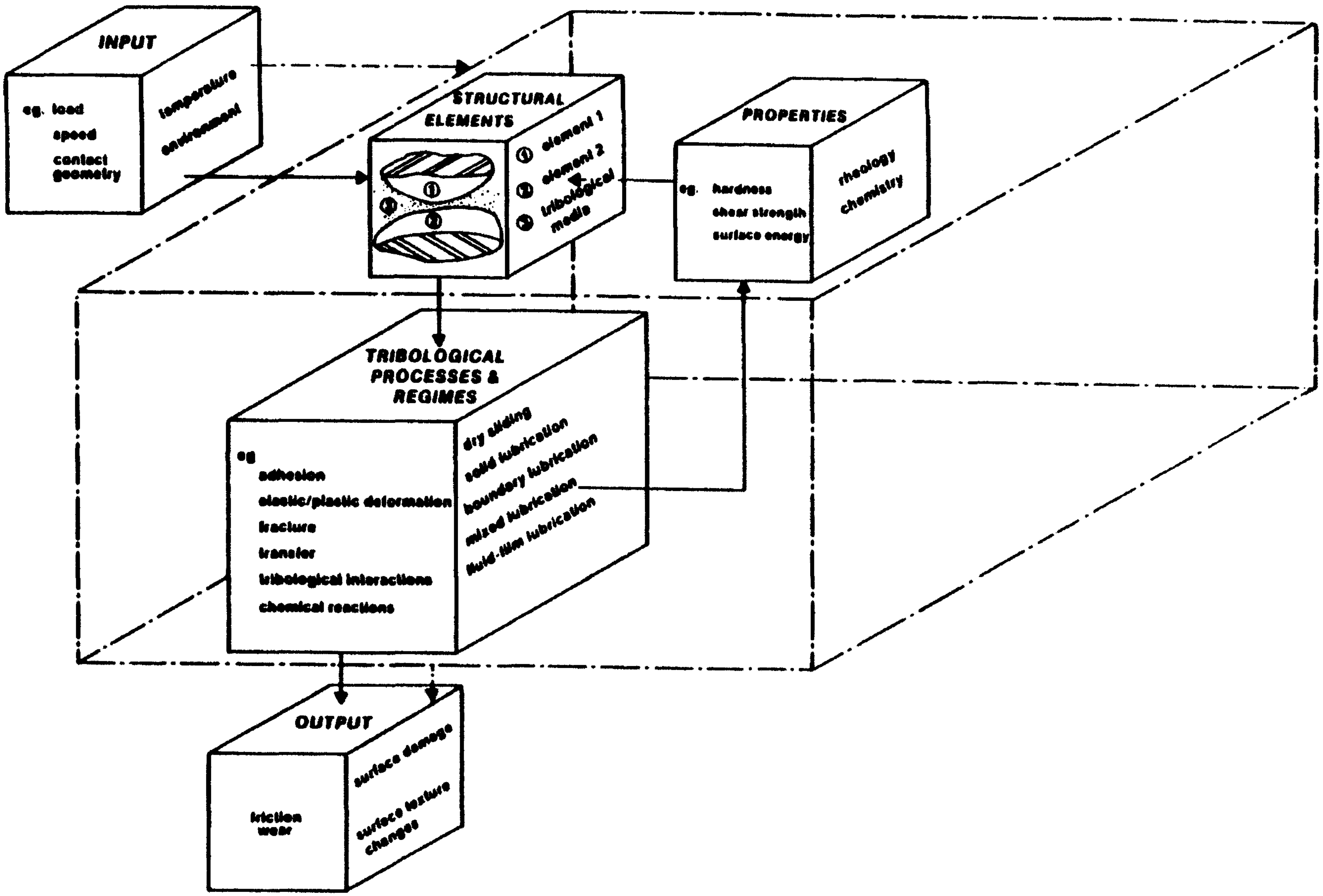

Figure 1. Schematic Diagram Showing Global Input–Output System in Tribology and Its Complex Sub–Systems.

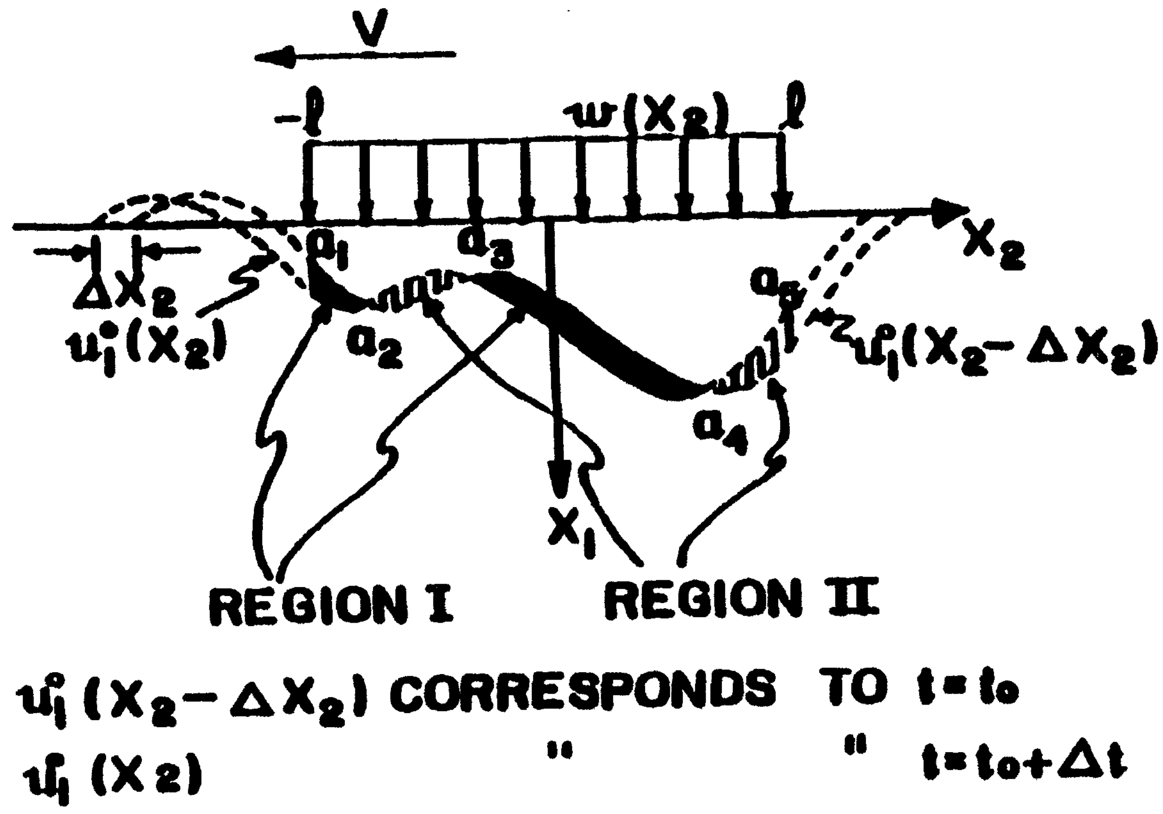

Figure 2. Schematic Diagram Depicting Deformation Friction of Viscoelastic Surface Films.

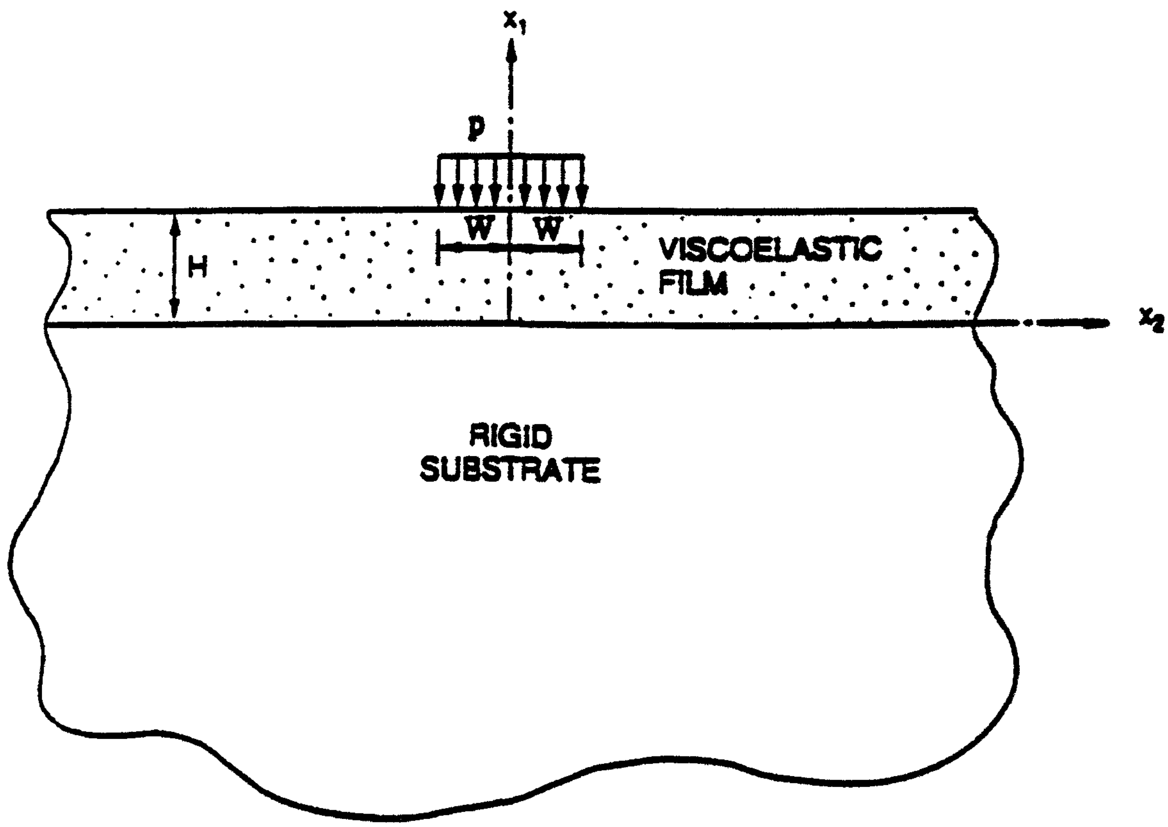

Figure 3. Schematic Diagram of the Model Details.

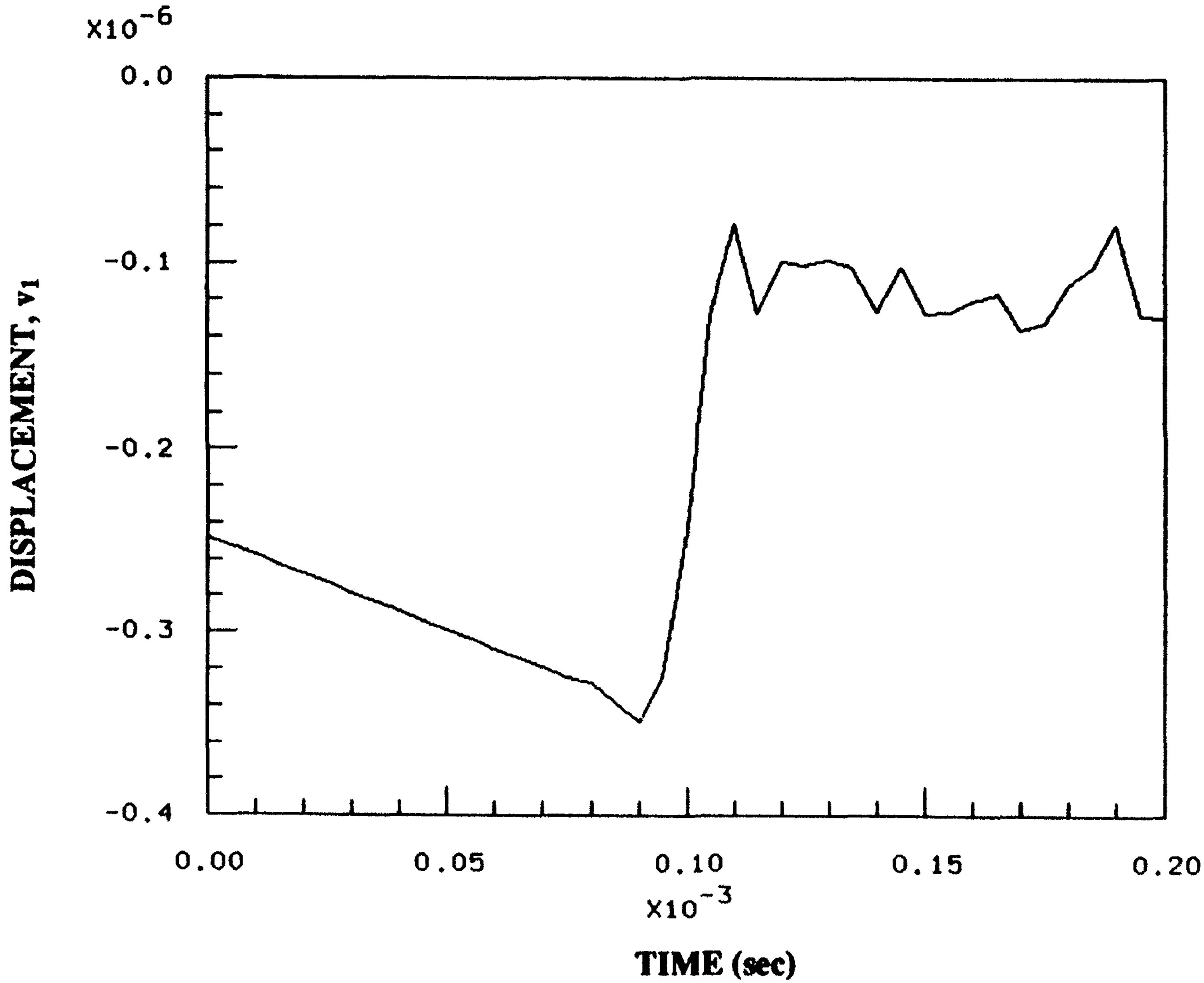

Figure 4. Surface Normal Displacement at the Center of the Loading Region During the Loading Period

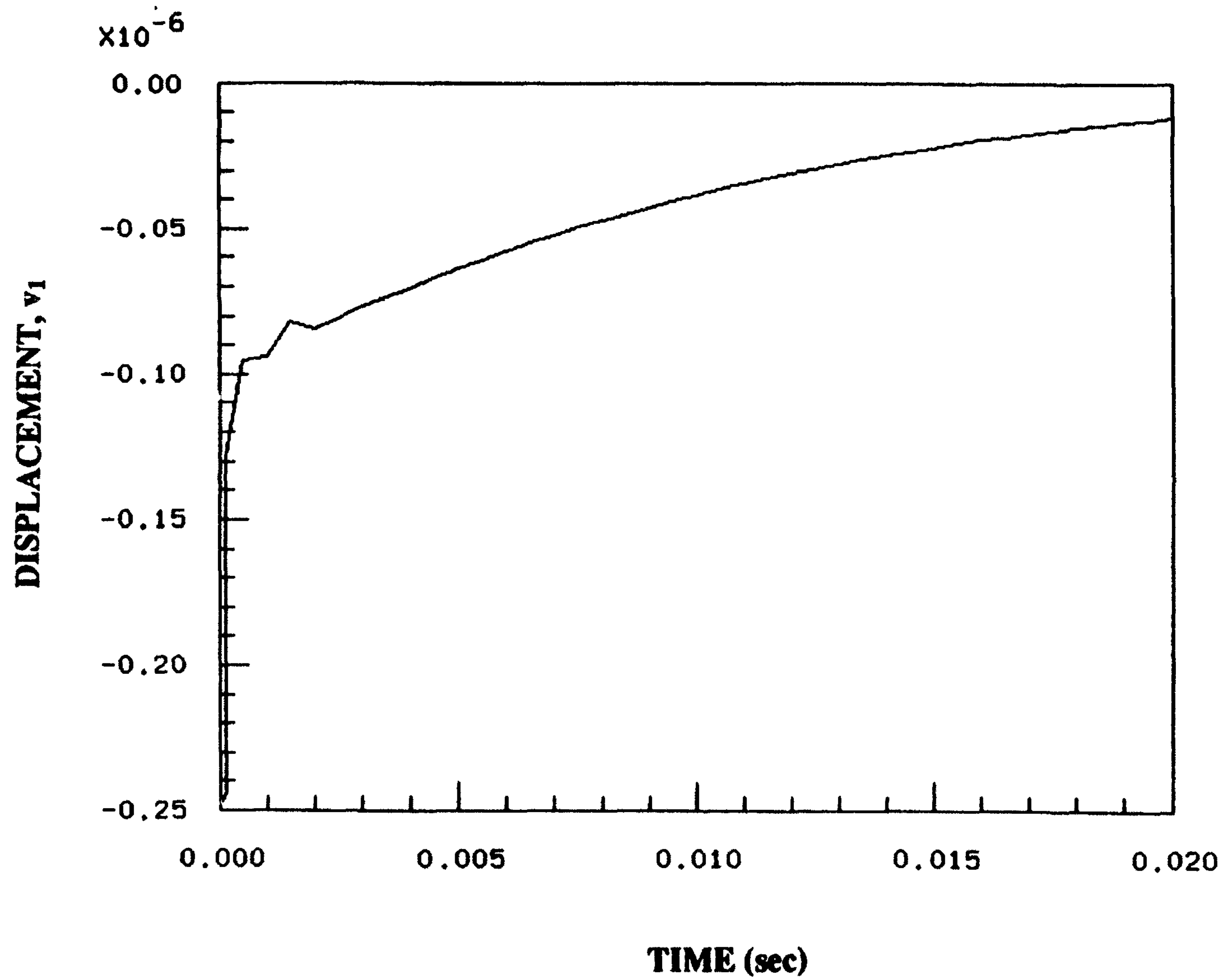

Figure 5. Surface Normal Displacement at the Center of the Loading Region After the Loading Period

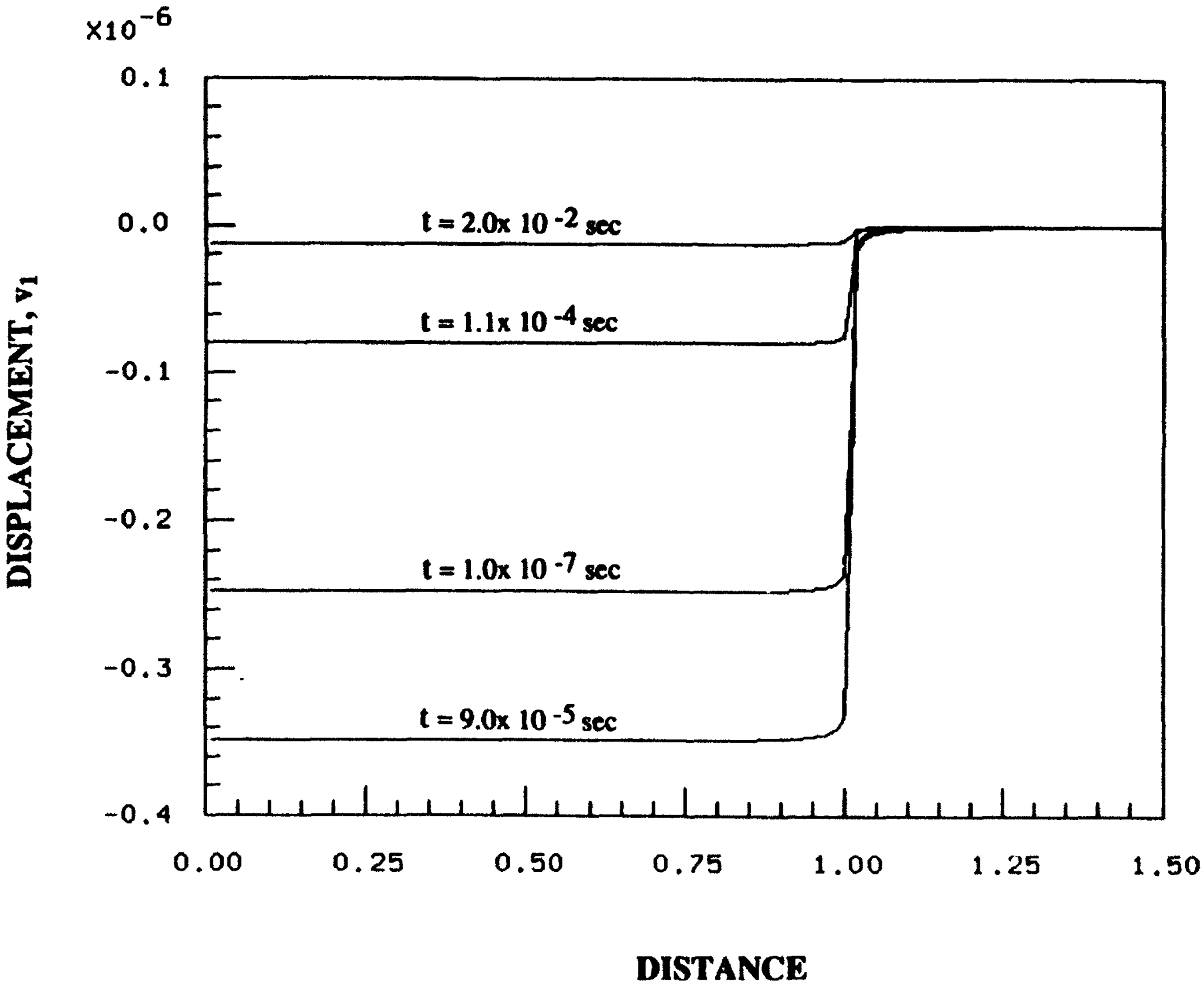

Figure 6. Surface Normal Displacement Over the Entire Loading Region

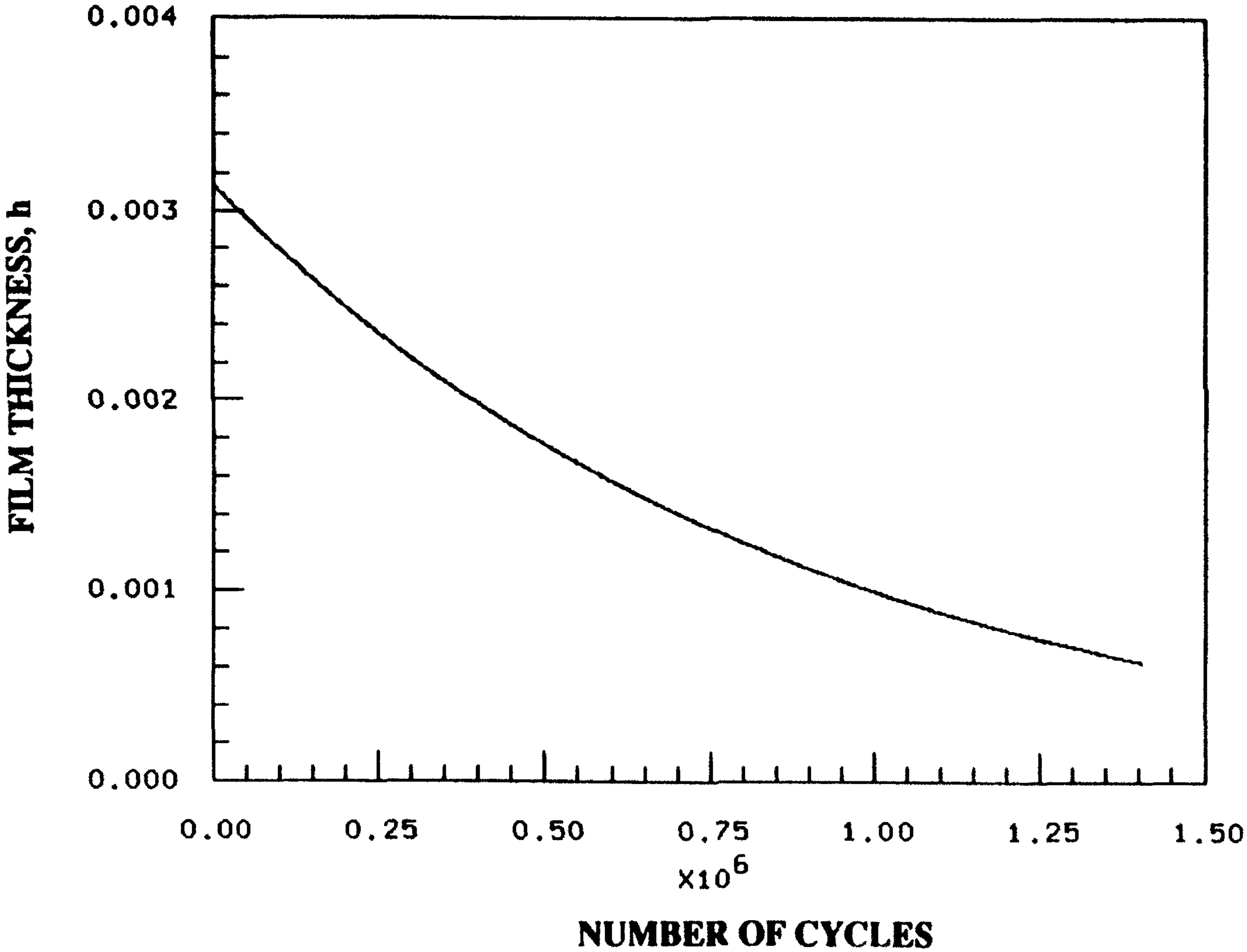

Figure 7. The Remaining Film Thickness Over the Entire Film Life

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Progress Report 0001AD
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on project:

ADVANCED MODEL OF GRADUAL WEAR OF VISCOELASTIC SURFACE FILM

Submitted to:

Office of Naval Research
Attn: Dr. Peter Schmidt
Materials Division/Code 1131
800 North Quincy Street
Arlington, VA 22217–5000

January 31, 1993

During this reporting period, a method of verifying the results experimentally, which is one of the tasks of the project, is proposed. To this end, the introduction of the mechanistic model in the project proposal is first recalled below:

A. Wear Criterion/Failure Rule – Consider a thin film on a substrate with certain surface roughness as characterized by the r.m.s. value, σ_0. Consider, moreover, an initial film thickness, $h_0/\sigma_0 > 1$, say 2 or more. Consider the wear as the gradual loss of material which comes under a slider repeatedly. The slider is under a normal load P and it is in relative motion with the film/substrate material system. The slow diminution of material will form the wear criterion and the associated failure rule is that $h = h_f$, where h_f is the film thickness at failure; this is a parameter relating to $h_0/\sigma_0 \simeq 1$.

B. Material System of the Model – Consider a deformable cylinder with certain surface roughness, rotating at constant speed, ω. Let there be a thin film overlaying the cylinder. As indicated above, the initial film thickness is h_0. The rationale is simply to capture the physics without unnecessarily making the analysis more complex than it needs to be. Various material constitutive relationships can be examined but, in this analysis, only the general linear viscoelastic material constitutive law will be studied.

C. Mechanics of the Proposed System – Models outlined above have the repeated loading feature built in. As such, it is an order of magnitude more complex than most contact problems posed in the literature. The models also would show the complex nature of an analysis to bring quantitative measure of wear into consideration. The choice of viscoelastic favored over plastic material models has to do with the fact that the loading and unloading of plastic bodies, aside from the simplest geometry, is extremely difficult to handle. More importantly, recent research has shown, under certain physical situations, viscoelastic description may be more appropriate.

The point to note is that plastic and viscoelastic are not mutually exclusive! With several material properties, the general linear viscoelastic model gives a relative measure of the viscousness and a relative measure of elasticity of the material. Anisotropic effects as well as microstructural effects all enter through the constitutive relationship formulation. In this formalism, the Newtonian framework on forces and stresses are set up independently from the material behavior. Material behaviors enter the problem through the formalism of constitutive relationship, e.g. stress–strain– strain rate law. The two sets of equations are then solved together with appropriate boundary and initial conditions.

There are still the questions of surface of the film and the slider geometry, among other factors, to be decided before a mechanics problem can be properly posed. We propose a uniform distribution of load under the slider. Also assumed is the negligible shear traction on the surface as we are addressing a solid lubricating system. The problem as posed is a transient problem as opposed to the usual steady state or quasi–steady state problems. The film thickness as a function of time must be found; this connotes wear rate. Also, the time at which the film thickness has diminished to a critical point must be found; this connotes failure. Of course, failure time determines wear life and the analysis is expected to provide wear life prediction. The associated stress fields, strain–rate (flow) fields and displacements on the surface will be part of the solution sought.

PROPOSED EXPERIMENT

Wear Apparatus

Shown schematically in Figure 1 is a generic pin–on–disk machine. This generic machine, which has the usual driving plate, loading arm, friction measuring device, must be augmented to form the proposed wear apparatus. It should be said here that the

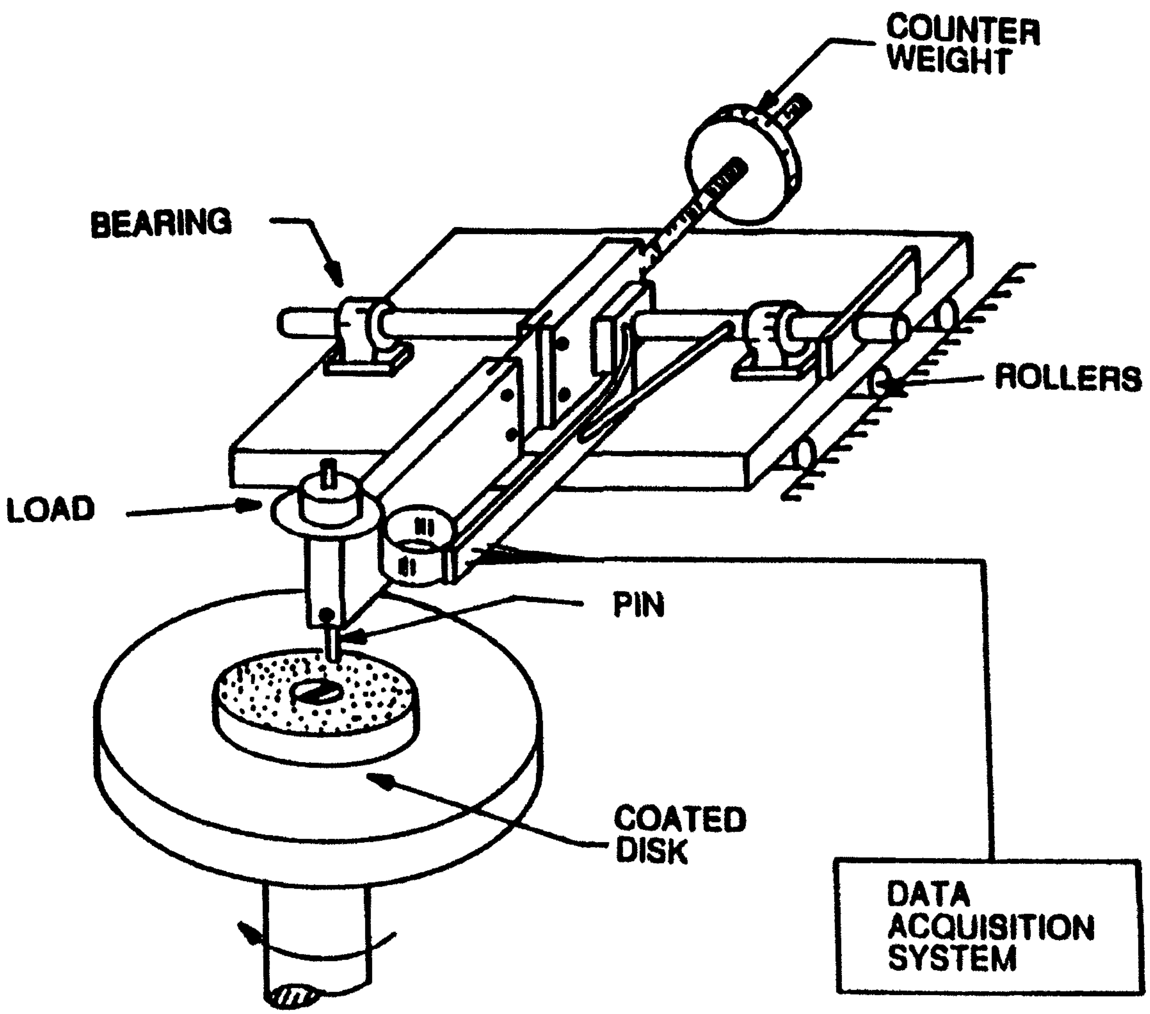

Figure 1. Schematic Diagram of a Pin-on-Disk Machine

pin–on–disk configuration is only one way to model mechanically the theoretical material system. The reason the pin–on–disk configuration is preferred is that there is a history of experience in the tribology field with this type of basic machine. For one thing, the rotating plate is the basic platform upon which the slider can be firmly anchored. The slider in this case is the coated disk. Also, the platform itself can be ordered so as to run true to its axis with little wobble or a controlled amount. The rider of the tribological pair is the pin.

Coated Disk

There are two parameters and a choice of viscoelastic coating material (lubricating) to be studied. The parameters are: (1) roughness of the disk; (2) thickness of the coating, assuming the sizes of the rotating plate and disk are predetermined. In the experiment envisioned, the roughness, σ_o, and initial thickness of the coating or film, h_o, should be varied as well as the ratio of h_o/σ_o.

The Pin

In order to simulate the mathematical model, the pin should be carefully designed and made, preferably of sufficiently hard material that would not change chemically in the presence of the coating. The geometry of the pin should be rectangular in cross–section with the dimension along the radial direction of the disk, when fixed, larger than the dimension perpendicular to the radial direction. After some trials, this ratio should be settled at, say, 2 to 1. The end should be crowned so as to give a uniform pressure according to Hertzian formula as an approximation of the current situation.

Measurement of Time

A counter, together with the speed and radius of the circular path, should yield the time of travel of the pin.

Measurement of Film Displacement

It is proposed that an instrument like the Photonic Sensor be used to measure the change of thickness of the film. Since the change per cycle is expected to be small, measurement should be taken at intervals.

Monitoring of Friction

Friction should be continuously monitored, and an alarm should be triggered when friction exceeds some pre-set level. Also, the alarm circuit should include shutting down the test. Barring catastrophic failure, the threshold of friction rise tells of the end of useful life of the film.

Materials Constants

As shown in Progress Report 0001AC, for the theory, four materials constants were used. For the proposed experiment, it is suggested that the viscoelastic film material also be a good "solid" lubricant. Otherwise, one of the assumptions concerning the low magnitude of shear traction as compared with normal traction under the load would not be valid. In fact, the whole thesis is on life prediction of lubricating film. This being the case, these constants may have to be measured inasmuch as solid lubricants vendors most likely do not provide what is called rheological data. Perhaps one of the questions would be: does one use the rheological properties of the lubricants as they are formulated or does one use the properties as "deposited" on the substrates? In the former case, a traditional method would be the vibratory excitation and response experiment. In the latter case, some kind of indentation experiment has to be designed.